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The Effect of the Depopulation Rate Parameters on the Remaining Atoms in the Bose-Einstein Condensation

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Abstract

The number of remaining atoms in the ^{87}Rb Bose-Einstein Condensation (BEC) is investigated in this paper with respect to the effect of the depopulation rate parameters that include the detuning frequency and the Rabi frequency, the point of extraction and the period of output coupling. This study is considered as an expanded study of the number of remaining atoms in relation to the effect of these parameters. The generalized rate equation obtained by F. Gerbier $et.al$ in 2001 have been modified and numerically solved. We have found an equation for calculating the amount of shift in the detuning frequency, which appears due to the absence of enough precise experimentally knowledge on the offset trapping potential. The comparison between our results with the available experimental data was satisfactory.

Keywords: Bose-Einstein Condensation, Atom laser, Quantum optics, Matter wave.

الملخص: في هذا البحث تم دراسة عدد الذرات المتبقية في تكثيف بوز اينشتاين (BEC) بالنسبة لتأثير معاملات معدل الاضمحلال لذرات ^{87}Rb والتي تشمل تردد detuning وتردد رابي ونقطة الاستخراج وفترة اقتران الخرج. هذه الدراسة تعتبر دراسة جديدة وموسعة لعدد الذرات المتبقية بالنسبة لتأثير هذه المعاملات. تم تعديل معادلة معدل الاضمحلال المعممة والتي حصل عليها F.Gerbier وآخرون في عام 2001 وحلها عدديًا. ولقد أوجدنا معادلة لحساب مقدار الازاحة في تردد detuning والذي يظهر بسبب عدم وجود معرفة تجريبية دقيقة كافية حول جهدoffset trapping. كانت المقارنة بين نتائجنا مع البيانات التجريبية المتوفرة مرضية.

1 Introduction

Samples of atoms with a macroscopic population in the ground state of the atomic gases are provided through the realization of the Bose-Einstein condensation (BEC) [1, 2, 3]. This population forms a coherent matter wave and is described by a macroscopic wave function[1, 4], which is the solution of the nonlinear Schrodinger equation [5]. The analogies between coherent matter waves and coherent photons lead to work out the theory for an atom laser[6, 7]. When the radiation frequency(rf) is applied to the condensation, the atom laser is produced and therefore both the number of remaining atoms in the BEC and the size of the BEC decrease as well [9]. The number of remaining atoms in the ^{87}Rb Bose-Einstein Condensation represents the main object of this study. The generalized rate equation introduced by F.Gerbier $et.al$ in 2001[10] have been modified and applied to find the number of remaining atoms in the BEC using the numerical solution. Accordingly, we study the change in the number of remaining atoms in the BEC with respect to the Rabi frequency and the detuning frequency that were previously studied in addition to the point of extraction and the period of output coupling at different values for each other for $F = 1,2$ hyperfine levels. To our best knowledge, there are no comprehensive and accurate studies dealing with the effect of the depopulation rate parameters on the number of remaining atoms in the BEC. In this paper we have studied the effect of the depopulation rate parameters on the remaining atoms in the BEC. The results have been compared with the available experimental results which show a satisfactory consistency.

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2 The System Wavefunction Formulation

The Bose-Einstein condensate (BEC) of dilute alkali vapors represents the main component of the atom laser which acts as the lasing mode, where the output beam is coupled to the lasing mode via the transfer of the internal state of atom from trapped into untrapped state[11]. Considering the system consists of the ^{87}Rb atoms which can develop a macroscopic population of the lowest energy state below a critical temperature T_c [6]. The atoms are confined in a harmonic magnetic potential:

$$V_{trap} = \frac{1}{2}M(\omega_x^2 x^2 + \omega_\perp^2 y^2 + \omega_\perp^2 z^2). \tag{1}$$

where M is the atomic mass and $\omega_x, \omega_\perp$ are frequencies of trap. A radio frequency magnetic field B_{rf} can induce transitions between states of hyperfine-manifold $F = 1\ or\ F = 2$. The $2F + 1$ Zeeman sublevels are represented by time-dependent macroscopic wavefunctions:

$$\Psi_m = \Psi'_m\, exp[-i\, m\, \omega_{rf}\, t], \tag{2}$$

where $m \in \{-F, \ldots, F\}$ and ω_{rf} is the frequency of B_{rf}. Often in experiments, Ioffe-type traps are usually elongated in the horizontal plane being axial symmetric in the remaining directions. Considering $y\ and\ z$ are the directions of radial confinement and x is axial confinement. In this system, the outer atoms based on rf-outcoupling can be described by generalized Gross−Pitaevskii equations(GPE) in $3D$ as following[5, 8]:

$$i\hbar\, \partial\Psi_m(\mathbf{r}, t)/\partial t = \left[\frac{P^2}{2M} + V_{eff,m}(\mathbf{r}, t)\right]\Psi_m(\mathbf{r}, t) \tag{3}$$
$$+\hbar\, \Omega \sum_{m'} (\gamma_{m,m'+1} + \gamma_{m,m'-1})\Psi_{m'}(\mathbf{r}, t)$$

where

$$V_{eff,m}(\mathbf{r}, t) = sign(g_f)\frac{1}{2}m\, M\, \omega\, \mathbf{r}^2 + m\, \hbar\, \delta + Mgz + U||\Psi(\mathbf{r}, t)||^2, \tag{4}$$

and γ is the delta function. An energy term, $U||\Psi(z, t)||^2$ is expressed twice as the mean-field energy of the system and plays a fundamental role in its dynamics with the interaction coupling constant $U = 4\pi\hbar^2\, N(t)\, a/M$, where a is the s-wave scattering length and the total density devided by the initial number N of atoms in the system is denoted by $||\Psi(\mathbf{r}, t)||^2 = \sum_m |\Psi_m(\mathbf{r}, t)|^2$[12, 5, 8]. The rf outcoupler is described by the coupling constant:

$$\hbar\Omega = g_F\, \mu_{Bohr}\, B_{rf}/\sqrt{2}, \tag{5}$$

which denotes to the Rabi frequency gotting due to the field B_{rf} for a Lande factor $g_{F=1} = -1/2, g_{F=2} = 1/2$. This paper studied two states for producing the atom laser, the transition from the magnetically trapped $|1, -1\rangle$ state to the untrapped $|1{,}0\rangle$ state and the transitions from the magnetically trapped $|2{,}2\rangle$ state to the untrapped $|2{,}0\rangle$ via the $|2{,}1\rangle$ state. In the $F = 2$ hyperfine level, the focussing will be on the transition $|2{,}1\rangle \to |2{,}0\rangle$ state considering the transition $|2{,}2\rangle \to |2{,}1\rangle$ which is considered as intermediate state. The detuning from the bottom of the trap of two states:

$$\hbar\delta = sing(m)(V_{off} - \hbar\omega_{rf}). \tag{6}$$

Gravity causes a sag in the trapped condensate away from the trap centre at $z = 0$ which depends on M and is given by $z_{sag} = g/|m|\omega_\perp^2$ where g is the gravitational acceleration. The offset of the trapping potential is denoted by V_{off} for the $m = 0$ condensate in the geometrical center of the trap at $z = 0$[5, 10, 13, 8, 14]. The time-dependant wavefunction of atoms interacting with each other in the condensation or at the large interaction can be easily obtained within the formalism of the mean-field theory as following:

$$\Psi(\mathbf{r}, t) = \alpha(t)\Phi(\mathbf{r}, t)\, e^{-iE}, \tag{7}$$

where $\alpha(t) = \sqrt{N(t)/N}$ and $E = \int_0^t (\mu(t')/\hbar + \delta)dt'$, E is an energy of BEC. The time-dependent ground state of energy μ (the chemical potential) in the Thomas-Fermi (TF) approximation is

$$\Phi(\mathbf{r},t) = \sqrt{\frac{\mu}{U}[1-\tilde{r}_\perp^2-\tilde{z}^2]}, \tag{8}$$

where $\tilde{r}_\perp^2 = (x/x_0)^2 + (y/y_0)^2, \tilde{z}^2 = (z/z_0)^2$ and $x_0 = \sqrt{2\mu/M\omega_x^2}$ and $y_0 = z_0 = \sqrt{2\mu/M\omega_\perp^2}$, those represent dimensions of the condensation at the frequency of trap $\omega = (\omega_x\,\omega_\perp^2)^{1/3}$. The chemical potential is given by

$$\mu = \frac{\hbar\,\omega}{2}\left(\frac{15\,a\,N}{\sqrt{\hbar/M\,\omega}}\right)^{2/5}. \tag{9}$$

At the time dependence of chemical potential μ, each of the condensate ground state of wave function $\Phi(t)$ and the (BEC) dimensions x_0, y_0, z_0 decrease with $N(t)$.

3 Number of atoms of the system

Immediately when the output coupling rf-field is switched on at $t = 0$, the whole condensate undergoes an initial rapid oscillations damping through all the coupled states due to the escape of the untrapped atoms out of the trap under the influence of gravity. At the same time, other atoms move to the resonance point replacing the leaving atoms, until a quasi-stationary state is reached[15].

In this stage, the number of atoms remaining in the condensate is studied depending on the effect of the depopulation rate parameters in 1D(z direction). Changing the number of atoms in the condensate with regard to change time can be found from the rate equation:

$$dN/dt = -\Gamma(N)\,N, \tag{10}$$

where Γ is depopulation rate which is given by the Fermi golden rule and by neglecting the transverse kinetic energy it becomes:

$$\Gamma \approx \ A\ Max[0,1-(z_{res}/z_0)^2]^2, \tag{11}$$

where $A = \frac{15\,\Omega^2\,\pi\,\hbar}{16\,\Delta}$, $\Delta = 2\,M\,g\,z_0$ and z_0 is the dimension of the condensation in the z direction [10]. Since the intermediate state is imposed, the mutual interaction between these two bound states is neglected so the upper level acquires a decay rate Γ for the $|2,2\rangle$ state $\sim \Gamma/2$ for the $|2,1\rangle$ state. The extraction point can be rewritten based on chemical potential through neglecting the spatial variations of the mean field potential and considering them as a perturbation comparing with gravity. In this case, the extraction point becomes:

$$z_{res} = \frac{-\hbar\delta + \mu(t)}{M\,g}. \tag{12}$$

Through this equation, we have studied the number of remaining atoms theoretically and compared it with the practical points mentioned in the Ref.[10]. We have noted that there is a shift in the frequency of detuning which was mentioned in the Ref.[10] due to the absence of enough precise experimentally knowledge of the offset trapping potential V_{off}. For this reason, we have used Eq.(4) to derive the following equation:

$$z_{res} = \frac{-g\,M + \sqrt{M}\sqrt{g^2\,M + 2\,\hbar\,\delta\,\omega_z^2}}{M\,\omega_z^2}. \tag{13}$$

We have used this equation as a tool to calculate the amount of shift in the frequency of detuning and therefore to match it with practical point without shift. Accordingly, we used this shift in the frequency of detuning of Eq.(12) adopted on the chemical potential through which we can study to effect the depopulation rate parameters on the number of remaining atoms in the BEC. The energy of BEC will be approximately restricted to an interval $[\frac{-\Delta}{2}, \frac{\Delta}{2}]$, that can be deduced by the resonance condition for the detuning [10]:

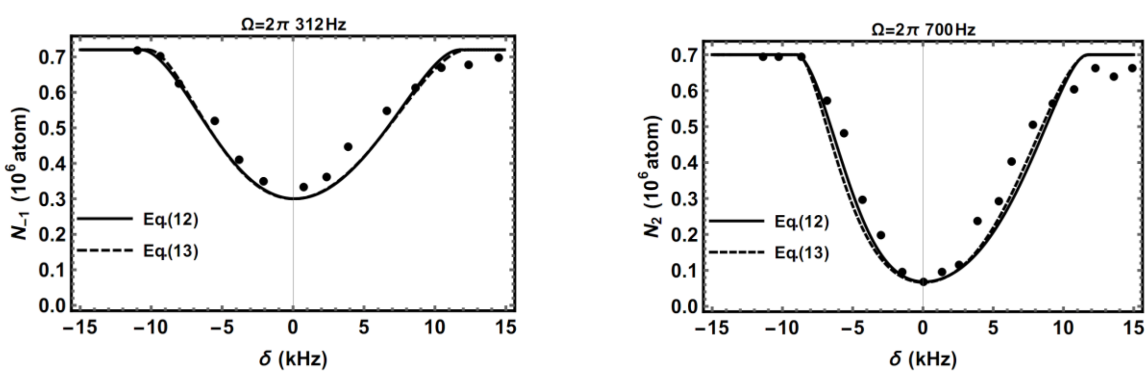
$$|\hbar\delta| \lesssim M\,g\,z_0. \tag{14}$$

Indeed, there are two regimes of the coupling which can be pointed out herewith. The first is the weak-coupling regime at $\Omega < \omega_z$ in the trapped atoms which are only disturbed slightly by the outcoupling process. The second is the strong-coupling regime for large Rabi frequencies in which the condensate shows a significant depletion[5]. The extraction process of atoms from condensate depends basically on both of the Rabi frequency and the period of output coupling. When the Rabi frequencies or the periods of output coupling are small, the number of outer atoms becomes small, and in this case redistribution of the remaining atoms in the condensate occurs, due to the movement of the higher energy atoms to the lower energy extraction region until the retraction of the condensate and the quasi-stationary state are reached. Based on the numerical solution of Eq.(10), the number of atoms remaining in the condensate can be deduced and subsequently the relation between the number of remaining atoms and the detuning frequency, point of the extraction, the Rabi frequency and period of rf output coupling can be investigated. Significantly, and upon studying the propagation of the atom laser, the effective parameters are really quantitative parameters which depend on a limited certain values of other parameters ($i.e$ they depend on the population of atoms in each region). In this paper, the typical values corresponding to the situation of Refs.[10, 9] have been taken: $N = 7.2 \times 10^5$ atoms initially, $\omega_x = 2\pi \times 13Hz$ and $\omega_\perp = 2\pi \times 140Hz$ for $|1,-1\rangle$ state while $|2,2\rangle$ state: $N = 7 \times 10^5$ atoms initially, $\omega_x = 2\pi \times 19Hz$ and $\omega_\perp = 2\pi \times 180Hz$.

4. Results and Discussion

4.1 The relation between the number of atoms remaining in the condensate and the depopulation rate parameters

Due to the shift existence in the frequency of detuning, the starting should be with studying the relation between the number of remaining atoms in the $|2,2\rangle$ and $|1,-1\rangle$ condensates $N_2\ and\ N_{-1}$ respectively and the detuning frequency shown in Fig.1. Whereas the number of remaining atoms decreases exponentially from the boundaries of the BEC (at positive and negative large values of the detuning) to the center of BEC at the detuning is almost zero. The curves shown in Fig.1 (a and b) illustrate the change of the number of remaining atoms with detuning frequency through the substitution of Eqs.(13 and 12) into Eq.(10) followed by its numerical solution. The points in this figure represent the experimental points obtained from Ref.[10]. From Fig.1 (a,b) we can see a good consistency (much better than Ref.[5]) with a slight difference.

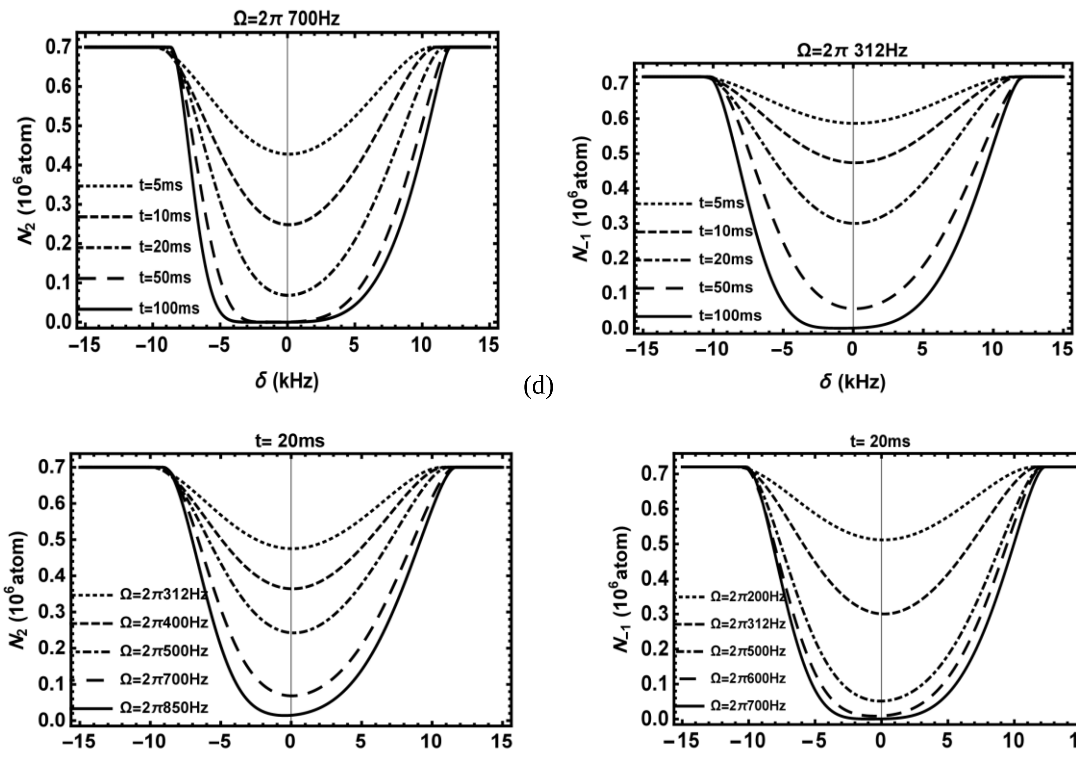


(a) (b)

Figure 1: The relation between the number of atoms in the condensate and the detuning frequency for $|2,2\rangle$ and $|1,-1\rangle$ states, (a, b). The points represent the expereimental resuts obtained from Ref.[10].

The curves that represent Eq.(13) in Fig.1 (a and b) conform with the experimental results (points) without any shift, while the curves of E(12) are plotted by adding a shift in the detuning in order to satisfy a curve conformation with the experimental points at the centre. There is an improvement coefficient in

the figures, its quantity is almost $1/2$ for $|2,2\rangle$ state and $|1,-1\rangle$ state.
On the other hand, the curves in Fig.2 (a and b) represent the behavior of the number of the remaining atoms (N_2, N_{-1}) with the detuning frequency at several values of the period of output coupling. Fig.2 (c and d) similarly do the same behavior but with several values of the Rabi frequency.

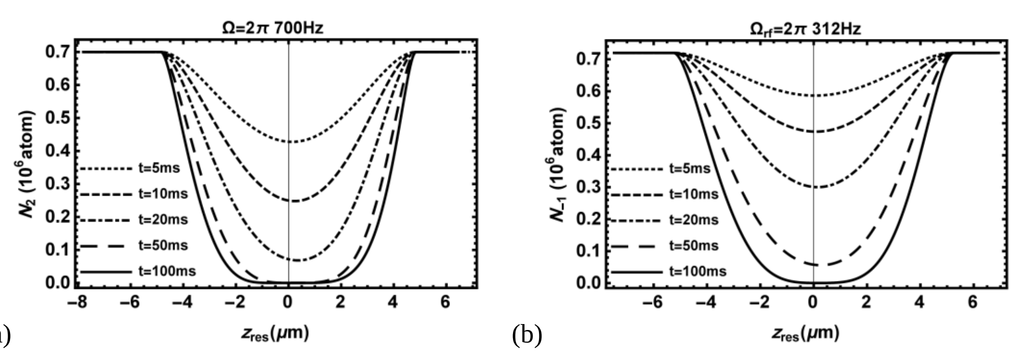


c) (d)

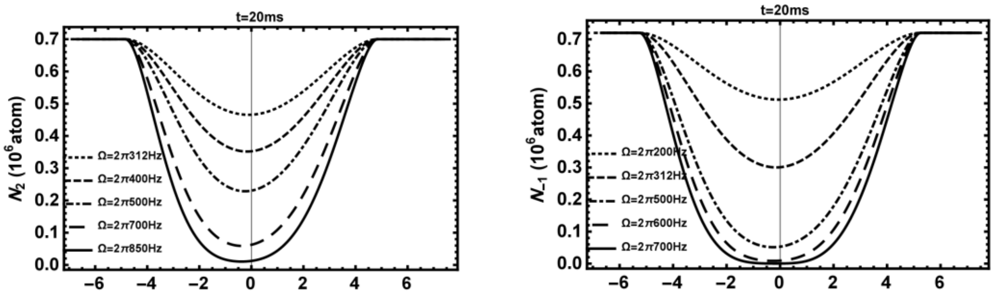
(e) (f)

Figure 2: The relation between the number of atoms in the condensate and the detuning frequency for $|2,2\rangle$ and $|1,-1\rangle$ states, (a, b) at certain values of period of output coupling and (e, f) at certain values of Rabi frequency.

The behavior of the number of remaining atoms in the condensate versus the point of extraction for $|2,2\rangle$ and $|1,-1\rangle$ states is displayed in Fig.3, where Fig.3(a, b) at certain periods of the output coupling and Fig.2(c, d) at certain values of the Rabi frequency. The number of atoms in the condensate is subjected to Thomas-Fermi distribution, so the number of remaining atoms decreases exponentially from the boundaries to the center of BEC, $i.e$, the center of BEC includes the largest number of atoms compared with other regions, this is displayed in all curves.



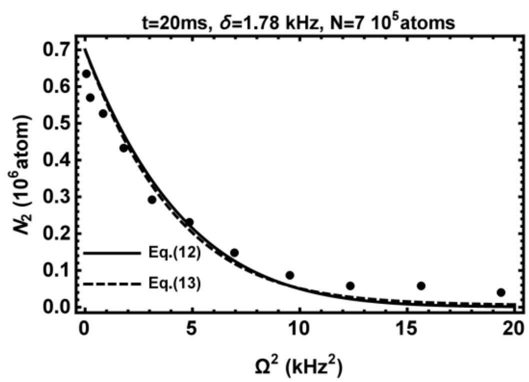
(a) (b)



(c) (d)

Figure 3: Presents the relation between the number of remaining atoms in the condensate with the extraction point for $|2,2\rangle$ and $|1,-1\rangle$ states. (a, b) at certain values of period of the output coupling, (c, d) at certain values of the Rabi frequency.

Fig.4 shows the relation between the number of the remaining atoms in the condensate for $|2,2\rangle$ and $|1,-1\rangle$ states versus the square of the Rabi frequency. Fig.4(a) clarifies the change of the number of remaining atoms in the $|2,2\rangle$ condensate after $20ms$ period of output coupling at the detuning $1.78kHz$ (it faces the rf at $1.736MHz$) with the square of the Rabi frequency depending on Eq.(13 and 12) as in Fig.1. While the points represent the experimental points obtained from Ref.[9]. The curves rather coincide with the experimental points. The plots of Fig.4 are plotted with the improvable coefficient $(1/2)$ of the two states in order to attain their coincidence with Fig.2. In Fig.4(b), the number of the remaining atoms in the $|2,2\rangle$ condensate (similarly, Fig.4(c) for the $|1,-1\rangle$ state) is plotted versus the square of the Rabi frequency after a $20ms$ period of output coupling at several values of the detuning frequency. Fig.4(d) shows this relation at $5kHz$ of the detuning frequency for certain periods of the output coupling (Fig.4(e) represents the $|1,-1\rangle$ state). From Fig.4, it is observed that the number of atoms changes as exponential decreasing function with square of the Rabi frequency. The condensate density subjects to Thomas-Fermi distribution TF[6], so that each region has a limited rate from the total number of BEC atoms. When the atoms are extracted from certain region of the BEC, the number of outer atoms increases until the atoms in that region finished. Accordingly, this explains the constancy of the curves at certain values of the number of remaining atoms versus the large values of the square of the Rabi frequency.



(a)

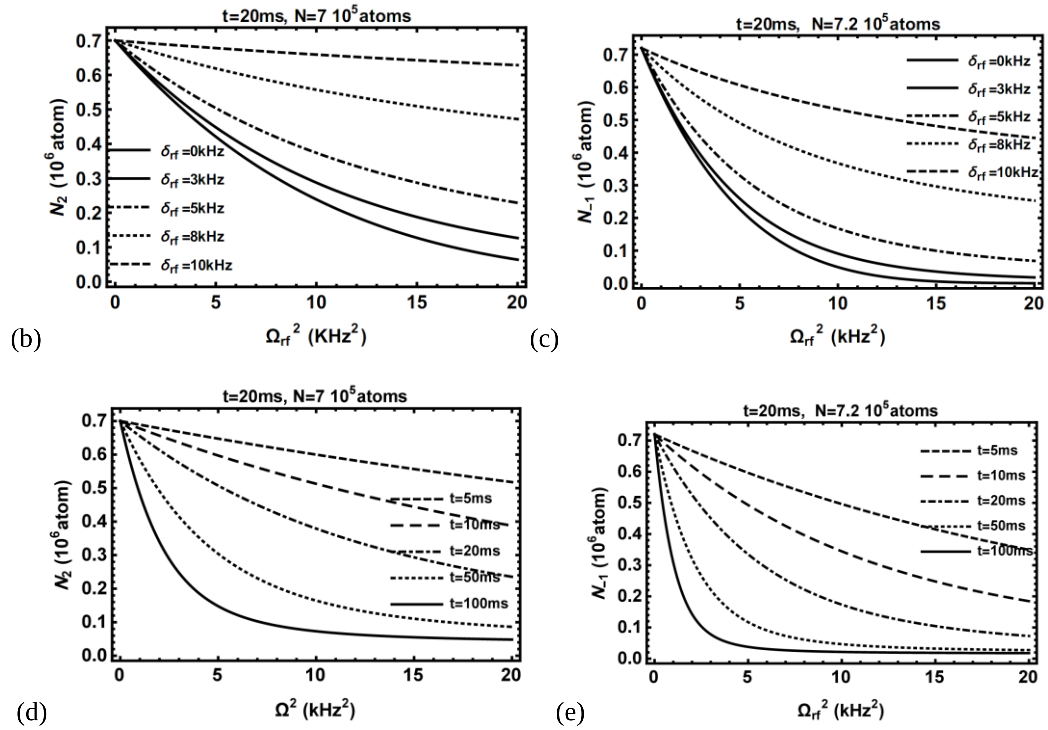


Figure 4: Illustrates the relation between the number of remaining atoms in the condensate with the square of Rabi frequency for $|2,2\rangle$ and $|1,-1\rangle$ states. (a) Comparison between the numerical solution of Eq.(10) and the experimental points obtained from [9]. (b, c) at certain values of detuning and (d, f) at certain values of period of output coupling.

Fig.5 shows the inversive relation between the number of remaining atoms and the period of output coupling. The depiction of Fig.5 is similar to that of Fig.4 but the effect of the square of the Rabi frequency on the atoms in the $|2,2\rangle$ state seems smaller than the effect of the period of output coupling.

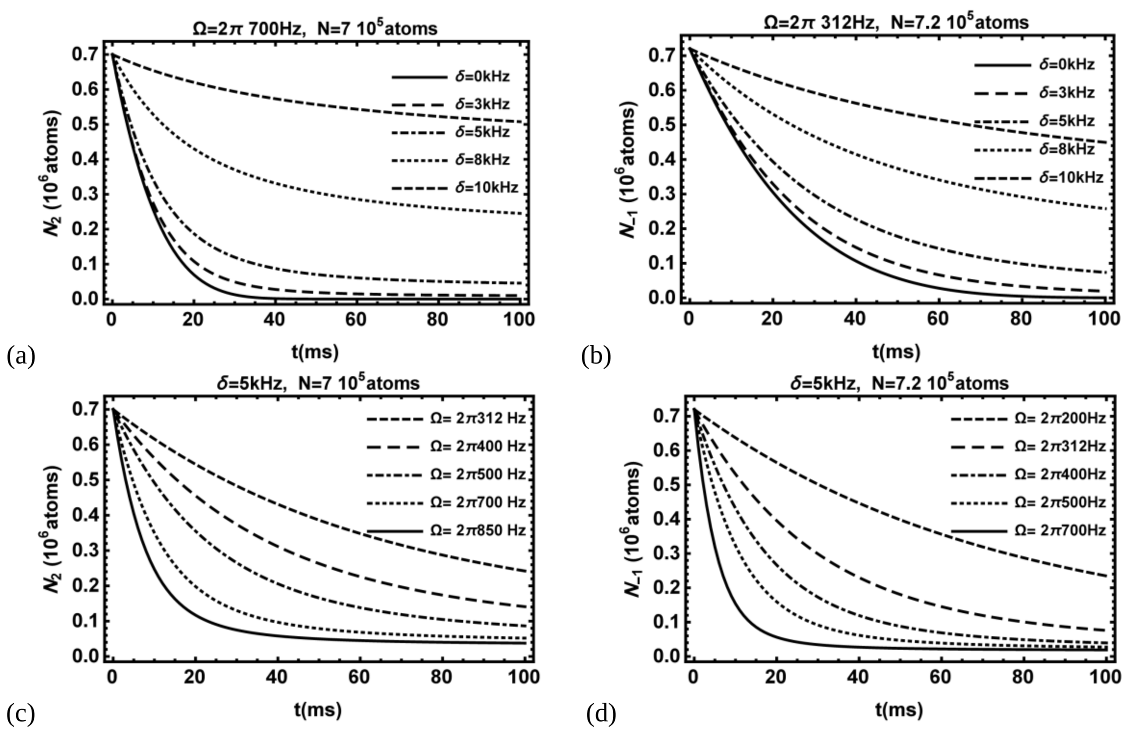


Figure 5: Shows the relation between the number of remaining atoms in the condensate with the period of output coupling for $|2,2\rangle$ and $|1,-1\rangle$ states. (a, b) at certain values of detuning and (c, d) at certain values of the square of Rabi frequency.

5. Conclusion

Our results show that the number of the remaining atoms in the condensate inversely proportional with the square of the Rabi frequency and the period of output coupling. This proportionality depends on the distribution of atoms in the condensate according to the Thomas-Fermi distribution. The direct proportionality between the number of remaining atoms and both of the detuning frequency and the extraction point according to the Thomas-Fermi distribution has been deduced also through the results. Through our study of the various values of the depopulation rate parameters, it can be concluded that there is a possibility for producing the best atom laser beam through controlling the number of the released atoms by determining the suitable values of the studied parameters. This is very useful to be mainly used in many wide applications such as atom optics, atom interferometers and precision measurements[16].

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